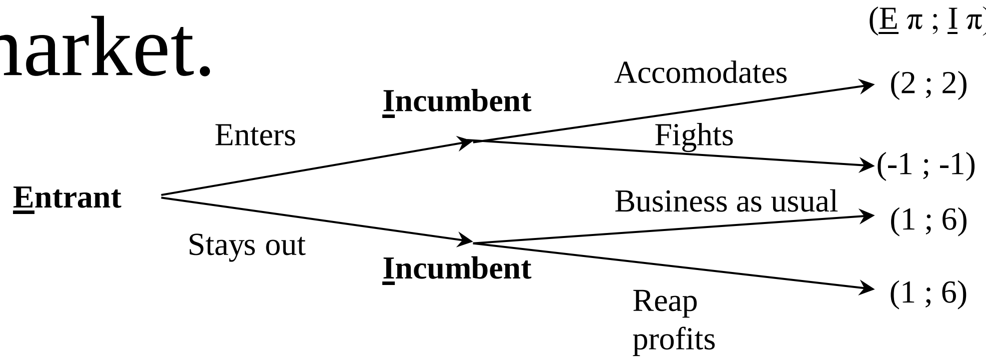
2nd Sample MT Exam

Lecture notes 4-7

Exercises 9-12

Start with the easiest! 11

- 11. There is currently one incumbent firm in a market, where a potential entrant considers entering the market.



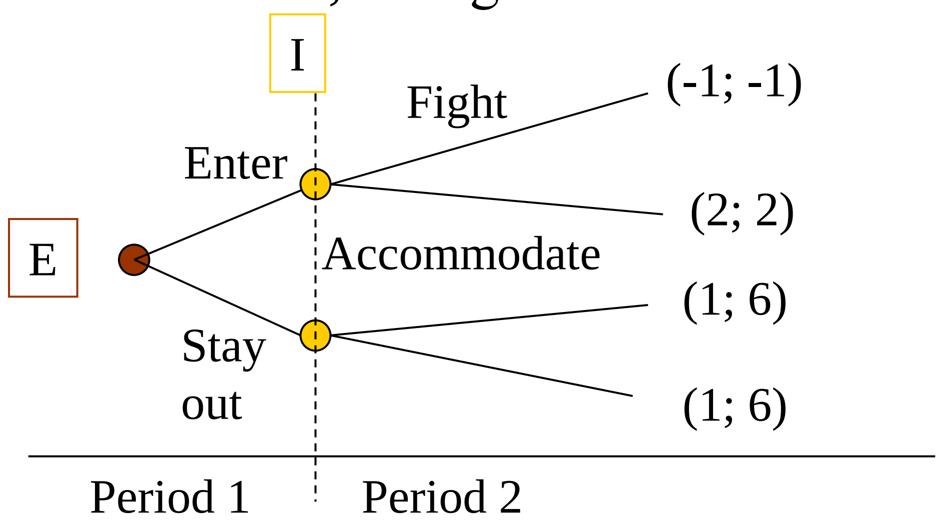
- Identify all subgame perfect Nash equilibria! Would the game have a different end result if the incumbent firm faced a similar "threat" (=a potential entrant) in 42 other markets?

Start with the easiest! 11

- The normal form of the game

		I	
		Fight/Reap profits	**A**ccommodate /Business as usual
E	**E**nter	-1; -1	2; 2
	Stay out	1; 6	1; 6

- It is different from one-period game in that firms select strategies rather than actions. For instance, I: “fight if E enters.” The extensive form:



Start with the easiest! 11

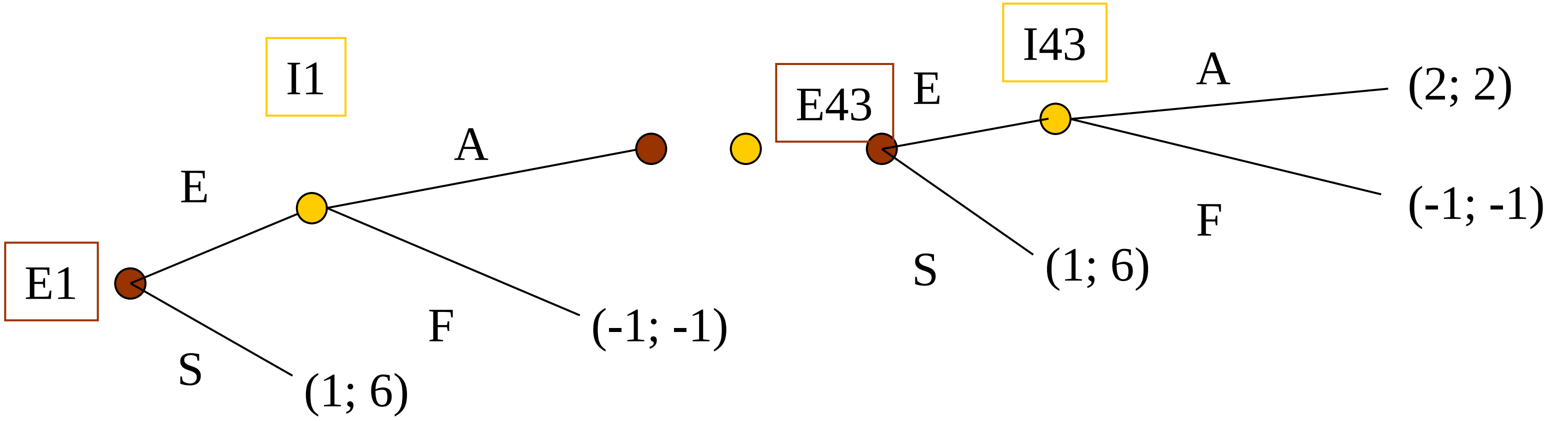
In a simultaneous game (normal form):

- If E = E and I = F, it is not a Nash equilibrium
- If E = E and I = A is a Nash equilibrium
- E = S and I = F is also a Nash equilibrium
- E = S and I = A is not a Nash equilibrium

- But equilibrium is defined in terms of strategies rather than actions!
- In case of E = S, I = F is just a threat from I to deter entry
- But is it credible to E?
- It is not, for it would not be in I’s interest to choose F if E = E
- In fact, I doesn’t even need to choose anything if E = S

Dynamic version

- To solve this problem work backwards!
- Since there are no more entrants to convince after market 43, E43 regards this game as a single interaction with I and E does not think that I = F is a credible strategy (as before)
- Hence, E = E and I = A (Nash equilibrium as before)
- The reasoning of E would be the same in market 42, market 41,…



Exercise 12

12. Eleven out of nineteen firms in an industry with Cournot competition would like to merge into one with a view to increasing their profits.

The demand curve is characterized by the equation $P=25000-Q$, while the total cost for all firms in the industry are given by the equation: $TC(q)=5000q+500000$.

Would the merger increase the profits of the merging companies compared to the pre-merger state of affairs if we assume that their costs would be characterized by the same cost function and they would still compete with the non-merged firms according to the rules of the Cournot model?

Horizontal mergers

- The most visible threat to competition
- The „merger paradox”: if firms do not merge to a monopoly, the merged firm may not be profitable
- Modeling the merger paradox: a Cournot model with N > 2 firms
  - Unit costs of the firms are identical: MC = c, FC > 0
  - $P = A - BQ = [A - B(Q_{-i} + q_i)]$; $Q_{-i} = Q - q_i$

$$\boxed{\pi_i(q_i, Q_{-i}) = [A - B(Q_{-i} + q_i) - c]q_i - FC_i; \quad \pi_i^C = \frac{(A-c)^2}{(N+1)^2 B} - FC_i; \quad q_i^* = \frac{A-c}{(N+1)B}}$$

  - Suppose that M < N firms merge: the industry will have N – M + 1 firms after the merger

The merger paradox (2)

- The profit functions of the merged (*m*) and the non-merged (*nm*) firms:

$$\pi_m^C(q_m, Q_{-m}) = [A - B(Q_{-m} + q_m) - c]q_m - FC_m; \quad Q_{-m} = \sum_{j=M+1}^{N} q_j$$

$$\pi_{nm}^C(q_i, Q_{-i}) = [A - B(Q_{-i} + q_{nm}) - c]q_{nm} - FC_{nm}; \quad Q_{-i} = \sum_{k=N-M}^{N} q_k + q_m - q_{nm}$$

- The merged firm becomes just like any of the other firms
- In Cournot-Nash equilibrium:

$$q_m = q_{nm} = \frac{A-c}{(N-M+2)B}; \quad \pi_m = \pi_{nm} = \frac{(A-c)^2}{(N-M+2)^2 B} - FC$$

- Profitable? Compare: $M\pi_i^C \leftrightarrow \pi_m$
- Merger is profitable, if $M\pi_i^C \leq \pi_m$

Exercise 10

10. Three firms that compete according to the rules of Cournot competition would like to form a cartel to increase their profits.

The demand curve is characterized by the equation $P=16000-2Q$, while the total cost for all firms are given by the equation: $TC(q)=1000q$.

Is it possible to sustain this cartel, if they would like to collectively earn as much profit as one monopolist and use trigger strategy to prevent deviations from the agreement, if the interest rate will be 25% and the probability of the collusion continuing for one more round is 80% in all rounds?

Cooperation versus conflict

- The Cournot-Nash equilibrium with N firms:

$$P = A - B\sum_i q_i \Rightarrow MR_i = A - BQ_{-i} - 2Bq_i = c \Rightarrow q_i^* = \frac{A-c}{2B} - \frac{(N-1)}{2} q \Rightarrow$$

$$\Rightarrow q^C = \frac{A-c}{(N+1)B}; \quad Q^C = \frac{N(A-c)}{(N+1)B} \Rightarrow \pi_i^C = \frac{(A-c)^2}{(N+1)^2 B}$$

- The cartel output of N firms with identical cost functions:

$$P = A - B\sum_i q_i = A - BNq \Rightarrow MR_i = A - 2BNq = c \Rightarrow$$

$$\Rightarrow q^M = \frac{A-c}{2NB} < q^C; \quad Q^M = \frac{(A-c)}{2B}; \quad \pi_i^M = \frac{(A-c)^2}{4NB} > \pi_i^C$$

- Cheating:

$$q^D = \frac{A-c}{2B} - \frac{(N-1)}{2} q^M \Rightarrow q^D = \frac{(N+1)(A-c)}{4NB};$$

$$Q^D = \frac{(3N-1)(A-c)}{4NB} \Rightarrow \pi^D = \left(A - BQ - c\right) q^D$$

Repeated games with infinite horizon

- Trigger strategy:
  - I will play the action on which we have agreed so long as you stick to our agreement
  - If you ever deviate from the agreement, I will play my punishment strategy forever
- Assume that the probability of the game continuing to the next period is ρ
- Trigger strategy of firm 2:
  - „I produce the cartel output (q^M) in period *t* if firm 1 has produced its cartel output in every previous period.
  - Should firm 1 produce more than its cartel output, I shall produce my Cournot-output (q^C) in every subsequent period."

Repeated games with infinite horizon

- The present value of firm 1's cooperative profit stream:

$$PV_{\infty}^{M}\left(\sum_{t=0}^{\infty}\pi_t^1\right)=\pi_0^1+\rho R\pi_1^1+(\rho R)^2\pi_2^1+\ldots+(\rho R)^t\pi_t^1+\ldots;$$

$$\text{Let }\ \Gamma=\rho R=\frac{\rho}{1+r}\Rightarrow PV^{\infty}\left(\sum_{t=0}^{\infty}\pi_t^1\right)=\frac{\pi_M^1}{1-\Gamma}$$

- The present value of firm 1's profit stream if it deviates:

$$PV_{\infty}^{D}\left(\sum_{t=0}^{\infty}\pi_t^1\right)=\left(\pi_0^1\right)^D+\rho R\left(\pi_1^1\right)^N+(\rho R)^2\left(\pi_2^1\right)^N+\ldots+(\rho R)^t\left(\pi_t^1\right)^N+\ldots;$$

$$\text{Let }\ \Gamma=\rho R=\frac{\rho}{1+r}\Rightarrow PV_{\infty}^{D}\left(\sum_{t=0}^{\infty}\pi_t^1\right)=\pi_D^1+\frac{\Gamma}{1-\Gamma}\pi_N^1$$

Repeated games with infinite horizon

- Compare the two strategies

$$\boxed{\begin{aligned}&PV_{\infty}^{M}\left(\pi^{1}\right) > PV_{\infty}^{D}\left(\pi^{1}\right) \Rightarrow \frac{\pi_{M}^{1}}{1-\Gamma} > \pi_{D}^{1} + \frac{\Gamma}{1-\Gamma}\pi_{N}^{1} \Rightarrow \\ &\Rightarrow \Gamma > \frac{\pi_{D}^{1}-\pi_{M}^{1}}{\pi_{D}^{1}-\pi_{N}^{1}} \Rightarrow r < \frac{\pi_{M}^{1}-\left[\rho\pi_{N}^{1}+(1-\rho)\pi_{D}^{1}\right]}{\pi_{D}^{1}-\pi_{N}^{1}}\end{aligned}}$$

- The larger the probability ρ and/or the smaller the interest rate r, the more probable it is to sustain the cartel

9. A potential entrant considers entering an industry that is currently served by a monopolist.

The demand curve is characterized by the equation $P=2025-10Q$, while the total cost of the incumbent is given by the equation: $TC(q_I)=40q_I+500$, and the total cost of the entrant by the equation $TC(q_E)=25q_E+25000$.

Determine the limit price and quantity that the incumbent firm could set to pre-empt entry! Would preventing the entry be profitable for the incumbent in the long run?

The Stackelberg model with different *MC*s

- The follower's best response function:

$$\boxed{\begin{aligned} &MR_2\left(q_1, q_2^*(q_1)\right) = A - Bq_1 - 2Bq_2 = c_2 \Rightarrow \\ &\Rightarrow q_2^*(q_1) = \frac{A - c_2}{2B} - \frac{q_1}{2} \end{aligned}}$$

- The leader's best response function and the solutions for output:

$$\boxed{\begin{aligned} &MR_1\left(q_1, q_2^*(q_1)\right) = A - Bq_2 - 2Bq_1 - B \cdot \frac{dq_2(q_1)}{dq_1} \cdot q_1 = c_1 \Rightarrow \\ &\Rightarrow q_1^* = \frac{2(A - c_1)}{3B} - \frac{2q_2}{3} = \frac{2(A - c_1)}{3B} - \frac{2}{3}\left(\frac{A - c_2}{2B} - \frac{q_1}{2}\right) = \\ &= \frac{A - 2c_1 + c_2}{2B}; \quad q_2^* = \frac{A + 2c_1 - 3c_2}{4B} \end{aligned}}$$

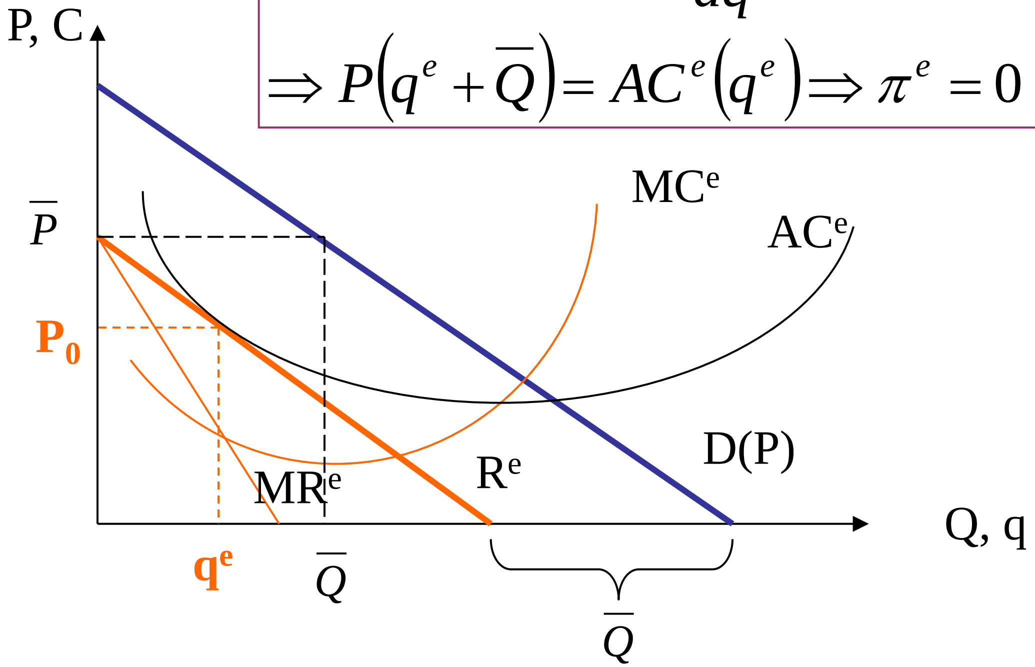
Limit pricing (1)

- The traditional models: Bain (1956) and Sylos-Labini (1962)
- The essence of the strategy: a simple Stackelberg model
- "Limit output" model: the quantity of output of the dominant firm affects the industry price = limit price
- The incumbent is the Stackelberg leader
- The entrant believes that its output choice will not alter the original output decision of the incumbent: the leader is irrevocably committed to its output choice
- The entrant's average cost declines over the initial range of production
- The incumbent can manipulate the entrant's profit calculation by the right choice of its output

Limit pricing (2)

- If incumbent selects the right output level, the entrant faces the residual demand curve: $R^e(P) = D(P) - \overline{Q}$
- The entrant maximizes: $PR^e(P) - C^e(q^e)$

$$MR^e\left(q^e\right) = \frac{d\left[D(P)-\overline{Q}\right]\cdot P(Q)}{dq^e} = MC^e \Rightarrow$$
$$\Rightarrow P\left(q^e + \overline{Q}\right) = AC^e\left(q^e\right) \Rightarrow \pi^e = 0$$



Limit pricing (3)

- The success of predation (limit pricing) depends on the entrant's belief about the incumbent's commitment to its output level
- Bain and Sylos-Labini argument: it would be costly for the incumbent to alter its output choice. For the example in the sample exam:
  - $P = 2025 - 10(q_E + q_I)$; $TC(q_I) = 500+40q_I$; $TC(q_E) = 25000 + 25q_E$

(a) The entrant's best response function (provided that they produce $q_E > 0$):

$$P = 2025 - 10q_E - 10q_I \Rightarrow MR(q_E) = 2025 - 10q_E - 20q_I = 25 = MC(q_I) \Rightarrow$$

$$\Rightarrow q_E = 100 - \frac{1}{2}q_I$$

(b) Objective of limit pricing: $P = AC_E \rightarrow \pi_E = 0 \rightarrow P * q_E - TC(q_E) = 0$

$$\pi_E = [2025 - 10q_E - 10q_I - 25] \cdot q_E - 25000 = 0; \quad q_E = 100 - \frac{1}{2}q_I \Rightarrow$$

$$\Rightarrow q_I = 200 - 2q_E \Rightarrow \pi_E = [2000 - 10q_E - 10(200 - 2q_E)] \cdot q_E - 25000 = 0 \Rightarrow$$

$$\Rightarrow 10q_E^2 = 25000 \Rightarrow q_E = 50 \Rightarrow q_I = \underline{100} \Rightarrow P = \underline{1025}$$